

SALZBURG MATHEMATICS COLLOQUIUM

Winter 2024/25

Arne Winterhof (RICAM Linz)

„Legendre Pairs“

January 23, 2025

Abstract:

Let $A = (a_0, a_1, \dots, a_{\ell-1}) \in \mathbb{C}^\ell$ be a finite complex sequence of length ℓ periodically continued with period ℓ . The periodic autocorrelation function $PAF(A, s)$ of A at lag s is $PAF(A, s) = \sum_{j=0}^{\ell-1} a_j \overline{a_{j+s}}$, $s = 0, 1, \dots, \ell-1$. Two sequences A and B of the same length ℓ form a Legendre pair (A, B) if $PAF(A, s) + PAF(B, s) = -2$, $s = 1, 2, \dots, \lfloor \ell/2 \rfloor$. Binary Legendre pairs of length ℓ , i.e. with $A, B \in \{\pm 1\}^\ell$, are pertinent to the construction of Hadamard matrices of order $2\ell + 2$ which can exist only if ℓ is odd. It is conjectured that there is a binary Legendre pair of every odd length ℓ . Several classes of Legendre pairs are known. The most prominent constructions are defined via characters of finite fields. In particular, there are Legendre pairs of length ℓ for every prime $\ell = p$ and every Mersenne number $\ell = 2^s - 1$, respectively. The smallest undecided case is $\ell = 115$. In the talk we summarize the known results on binary Legendre pairs. We then also discuss more recent work on quaternary Legendre pairs of length ℓ (joint work with Ilias S. Kotsireas and Christoph Koutschan), i.e. with $A, B \in \{\pm 1, \pm i\}^\ell$, and generalizations to k -ary sequences (joint work with Huaning Liu).

Thursday, 15:00-15:45

Hörsaal 414, 1. Stock