Fachbereich Mathematik
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Gastvortrag

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Seminarraum II
17 Uhr (c.t.)

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Properties of *N*-Continued Fraction Expansions

Einladender: Fritz Schweiger

PROPERTIES OF N-CONTINUED FRACTION EXPANSIONS

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In 2008, Ed Burger and his co-authors showed in [2] that every quadratic irrational number x has infinitely many N-continued fraction expansions of period length 1. Here, for every integer N different from 0, an N-continued fraction expansions of $x \in [0,1)$ is a continued fraction expansion of x of the form

$$x = \cfrac{N}{c_1 + \cfrac{N}{c_2 + \ddots + \cfrac{N}{c_n + \ddots}}}$$

where the partial quotients c_i are integers (which are positive if $N \geq 1$). Note that if $N = 1$ the N-continued fraction expansion is the classical regular continued fraction expansion (RCF). Last year, Maxwell Anselm and Steven Weintraub showed in [1] that in fact every $x \in (0,1)$ has infinitely many N-continued fraction expansions for every $N \geq 2$ fixed. There is one special case, in which the partial quotients c_i are always at least of size N. This special case (which Anselm and Weintraub call *best* cf_N-expansions), which can be seen as an analogue of the RCF, has properties which are very similar to those of the RCF, and properties which seem surprisingly different. For example, in this talk I will show that for these best cf_N-expansions there is an underlying Gauss-map, and for this map the invariant measure is known. However, Anselm and Weintraub conjecture that there are $N \geq 2$ and quadratic irrationals for which the corresponding best cf_N-expansions do not have an ultimately periodic expansion. Further support for this conjecture will be given in this talk. Finally, a variation of the N-continued fraction expansion will be discussed which yields continued fraction expansions of $x \in (0,1)$ of the form

$$x = \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \ddots + \cfrac{a_n}{b_n + \ddots}}}$$

where the a_i and b_i are positive integers, and $b_i \in \{a_i^2, a_i^2 + 1, \ldots, a_i^2 + a_1 - 1\}$. The underlying Gauss-map of this expansion will be given. Unfortunately, most properties of this expansion (invariant measure, etc.) are still unknown.

References

[1] Anselm, M. and Weintraub, S.H. – *A generalization of continued fractions*, J. Number Theory **131** (2011), no. 12, 2442-2460

[2] Burger, E.B., Gell-Redman, J., Kravitz, R., Walton, D., Yates, N. – *Shrinking the period lengths of continued fractions while still capturing convergents*, J. Number Theory **128** (2008), no. 1, 144–153.